LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2023**

UMT 4403 – MATHEMATICS FOR STATISTICS - II

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions (10 x 1 = 10)
1.	**Answer the following**
a)	Define bounded sequence.
b)	What is conditional convergence of a series?
c)	When do we say that a function is strictly increasing?
d)	What is a derivative of a function at a point?
e)	Define measure zero.
2.	**Fill in the blanks**
a)	If $A \subset B$ and $B \subset A$ then we can say A___B.
b)	An alternate series is an infinite series whose terms alternate in __________.
c)	The oscillation of f over $J, \omega\,[f, J] =$ _____________.
d)	Derivative of a constant function is __________.
e)	Every countable subset of $\mathbb{R}$ has measure _______.
	SECTION A - K2 (CO1)
	Answer ALL the Questions (10 x 1 = 10)
3.	**MCQ**
a)	The sequence $\left\{\frac{1}{n}\right\}$ converges to ______. 1. 0 2. 1 3. ¼ 4. Does not Converge.
b)	The series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ is ______ series. 1. Conditionally Convergent 2. Divergent 3. Convergent 4. None of these
c)	$f(x) = 2x$ and $g(x) = 0$. Then which of the following is not true. 1. $f + g$ is continuous 2. $f - g$ is continuous 3. fg is continuous 4. $\frac{f}{g}$ is continuous
d)	A differentiable function is __________. 1. discontinuous 2. Discontinuous at only one point 3. Continuous 4. None of these
e)	$\int \cos x\, dx =$ ______ 1. $\sin x$ 2. $-\sin x$ 3. $\cot x$ 4. None of these.
4.	**True or False**
a)	Every increasing sequence with upper bound converges to a limit point.
b)	Every absolutely convergent series is conditionally convergent.
c)	Let $f: \mathbb{R} \to \mathbb{R}$ and $a \epsilon \mathbb{R}$ .If f is continuous at a, then $\omega[f, a] = 0$.
d)	Let $f: \mathbb{R} \to \mathbb{R}$, then $f(x) = 2x$ is a differentiable function.

e)	Integration can be used to calculate area.
SECTION B - K3 (CO2)	
	Answer any TWO of the following **(2 x 10 = 20)**
5.	If $\{s_n\}_{n=0}^{\infty}$ is a sequence of real numbers which converges to L, then show that $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2
6.	If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B then show that $\sum_{n=1}^{\infty} a_n + b_n$ converges to $A+B$ and $\sum_{n=1}^{\infty} ca_n$ converges to cA where $c\epsilon\mathbb{R}$
7.	If f is a non-decreasing function on the bounded open interval (a,b) and f is bounded above on (a,b) then show that $\lim_{x\to b^-} f(x)$ exist also if f is bounded below on (a,b) then show that $\lim_{x\to a^+} f(x)$ exist.
8.	If f and g both have derivatives at $c \in \mathbb{R}$,then show that $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$ and $(f+g)'c = f'(c) + g'(c)$
SECTION C – K4 (CO3)	
	Answer any TWO of the following **(2 x 10 = 20)**
9.	a.) If $\{s_n\}_{n=0}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} s_n = L$ then prove that $L \geq 0$ **(8 Marks)** b.) Determine the limit of the sequence $\left\{\frac{1}{3n^2}\right\}_{n=0}^{\infty}$ **(2 Marks)**
10.	Demonstrate that the series $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{1-x}$ if $0 < x < 1$ and diverges if $x \geq 1$ and hence determine the $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent or divergent.
11.	Categorize the continuous and discontinuous function among the following. 1. sin x 2. $f(x) = \begin{cases} 0 & if\ x = 0 \\ 1 & otherwise \end{cases}$ 3. $f(x) = \|x\|$ 4. $f(x) = \frac{1}{x}$ 5. $f(x) = \sin(\cos(x))$ 6. $f(x) = \sin x^2$ 7. $f(x) = \cos^2 x$ 8. $f(x) = 2$ 9. $f(x) = \frac{1}{x+1}$ 10. $f(x) = x^2$
12.	State and prove Rolle's theorem.
SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 20 = 20)**
13.	a.) Determine the following sets of points in the plane geometrically. 1. A=$\{(x,y)\|x^2 + y^2 = 1\}$, 2. B=$\{(x,y)\|x \leq y\}$, 3. C=$\{(x,y)\|x + y = 2\}$ **(5 Marks)** b.) "If A and B are subsets of S, then $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$" – justify your answer. **(15 Marks)**
14.	Explain about alternating series and hence prove that, if $\{a_n\}_{n=0}^{\infty}$ sequence of a positive numbers such that 1.) $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq \cdots$ 2.) $\lim_{n\to\infty} a_n = 0$ then the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
15.	State and Prove Taylor's Formula.
16.	"If $f \in \mathbb{R}[a,b], g \in \mathbb{R}[a,b]$ $f+g \in \mathbb{R}[a,b]$ and $\int_a^b f+g = \int_a^b f + \int_a^b g$"- Justify your answer.

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